Neutrino oscillation, SUSY GUT and B decay

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Abstract

Effects of supersymmetric particles on flavor changing neutral current and lepton flavor violating processes are studied in the supersymmetric SU(5) grand unified theory with right-handed neutrino supermultiplets. Using input parameters motivated by neutrino oscillation, it is shown that the time-dependent CP asymmetry of radiative B decay can be as large as 30% when the $\tau \to \mu\,\gamma$ branching ratio becomes close to the present experimental upper bound. We also show that the B_s–$\bar{B}_s$ mixing can be significantly different from the presently allowed range in the standard model.

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Although the standard model (SM) of the elementary particle theory describes current experimental results very well, particles and interactions outside of the SM may appear beyond the energy scale available at current collider experiments. One of indications is already given by the atmospheric and the solar neutrino anomalies which have been interpreted as evidences of neutrino oscillation [1,2]. A natural way to introduce small neutrino masses for the neutrino oscillation is the see-saw mechanism [3] where the right-handed neutrino is introduced with a very heavy mass. This scenario suggests the existence of a new source of flavor mixings in the lepton sector at much higher energy scale than the electroweak scale.

In this letter we consider flavor changing neutral current (FCNC) processes and lepton flavor violation (LFV) of charged lepton decays in the model of a SU(5) supersymmetric (SUSY) grand unified theory (GUT) which incorporates the see-saw mechanism for the neutrino mass generation. In the SUSY theory, the superpartners of quarks and leptons, namely squarks and sleptons respectively, have new flavor mixings in their mass matrices. In the model based on the minimal supergravity these mass matrices are assumed to be flavor-blind at the Planck scale. However renormalization effects due to Yukawa coupling constants can induce flavor mixing in the squark/slepton mass matrices. In the present model sources of the flavor mixing are Yukawa coupling constant matrices for quarks and leptons as well as that for the right-handed neutrinos. Because the quark and lepton sectors are related by GUT interactions, the flavor mixing relevant to the Cabibbo-Kobayashi-Maskawa (CKM) matrix can generate the LFV such as $\mu \to e\,\gamma$ and $\tau \to \mu\,\gamma$ processes [4] in addition to FCNC in hadronic observables [5]. In the SUSY model with right-handed neutrinos, it is possible that the branching ratios of the LFV processes become large enough to be measured in near-future experiments [6]. When we consider the right-handed neutrinos in the context of GUT, the flavor mixing related to the neutrino oscillation can be a source of the flavor mixing in the squark sector. We show that due to the large mixing of the second and third generations suggested by the atmospheric neutrino anomaly, B_s–$\bar{B}_s$ mixing, the time-dependent CP asymmetry of the $B \to M_s\,\gamma$ process, where M_s is a CP eigenstate including the strange quark, can have a large deviation from the SM prediction.

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The Yukawa coupling part and the Majorana mass term of the superpotential for the SU(5) SUSY GUT with right-handed neutrino supermultiplets is given by

$$W = \frac{1}{8} f_U^{ij} \Psi_i \Psi_j H_5 + f_D^{ij} \Psi_i \Phi_j H_{\bar{5}} + f_N^{ij} N_i \Phi_j H_5 + \frac{1}{2} M_\nu^{ij} N_i N_j \; , \tag{1}$$

where Ψ_i, Φ_i and N_i are **10**, $\bar{\mathbf{5}}$ and **1** representations of SU(5) gauge group. $i, j = 1, 2, 3$ are the generation indices. H_5 and $H_{\bar{5}}$ are Higgs superfields with **5** and $\bar{\mathbf{5}}$ representations. In terms of the SU(3)×SU(2)$_L$×U(1)$_Y$, Ψ_i contains Q_i (**3**, **2**, $\frac{1}{6}$), U_i ($\bar{\mathbf{3}}$, **1**, $-\frac{2}{3}$) and E_i (**1**, **1**, 1) superfields. Here the representations for SU(3) and SU(2) groups and the U(1)$_Y$ charge are indicated in the parentheses. Φ_i includes D_i ($\bar{\mathbf{3}}$, **1**, $\frac{1}{3}$) and L_i (**1**, **2**, $-\frac{1}{2}$), and N_i is a singlet of SU(3)×SU(2)$_L$×U(1)$_Y$. M_ν is the Majorana mass matrix. Below the GUT scale ($\approx 2 \times 10^{16}$ GeV) and the Majorana mass scale ($\equiv M_R$) the superpotential for the minimal supersymmetric standard model (MSSM) fields is given by

$$W_{\mathrm{MSSM}} = \tilde{f}_U^{ij} Q_i U_j H_2 + \tilde{f}_D^{ij} Q_i D_j H_1 + \tilde{f}_L^{ij} E_i L_j H_1 + \mu H_1 H_2 - \frac{1}{2} \kappa_\nu^{ij} (L_i H_2)(L_j H_2) \; , \tag{2}$$

where κ_ν is obtained by integrating out the heavy right-handed neutrino fields. At the right-handed neutrino mass scale κ_ν is given as $\kappa_\nu^{ij} = (f_N^{\mathbf{T}} M_\nu^{-1} f_N)^{ij}$. The Yukawa coupling constants $\tilde{f}_U^{ij}$, $\tilde{f}_D^{ij}$ and $\tilde{f}_L^{ij}$ are related to the coupling constants f_U^{ij} and f_D^{ij} at the GUT scale. The quark, charged lepton and neutrino masses and mixings are determined from the superpotential Eq. (2) at the low energy scale.

As discussed above, the renormalization effects due to the Yukawa coupling constants induce various FCNC and LFV effects from the mismatch between the quark/lepton and squark/slepton diagonalization matrices. In particular the large top Yukawa coupling constant is responsible for the renormalization of the $\tilde{q}_L$ and $\tilde{u}_R$ mass matrices. At the same time the $\tilde{e}_R$ mass matrix receives sizable corrections between the Planck and the GUT scales and various LFV processes are induced. In a similar way, if the neutrino Yukawa coupling constant f_N^{ij} is large enough, the $\tilde{l}_L$ mass matrix and the $\tilde{d}_R$ mass matrix receive sizable flavor changing effects due to renormalization between the Planck and the M_R scales and the Planck and the GUT scales, respectively. These are sources of extra contributions to

LFV processes and various FCNC processes such as $b \to s\gamma$, the B^0–$\bar{B}^0$ mixing and the K^0–$\bar{K}^0$ mixing.

It is particularly interesting that the chiral structure of the FCNC amplitudes due to the $\tilde{d}_R$ mixing is different from that expected in the SM. For example, the flavor mixing in the $\tilde{d}_R$ sector generates a sizable contribution to the $b \to s\gamma_R$ amplitude through gluino–$\tilde{d}_R$ loop diagrams, whereas this amplitude is suppressed by a factor m_s/m_b over the dominant $b \to s\gamma_L$ amplitude in the SM. When the amplitudes with both chiralities exist, the mixing-induced time-dependent CP asymmetry in the $B \to M_s\gamma$ process can be induced. Using the Wilson coefficients c_7 and c_7' in the effective Lagrangian for the $b \to s\gamma$ decay $\mathcal{L} = c_7\bar{s}\sigma^{\mu\nu}b_R F_{\mu\nu} + c_7'\bar{s}\sigma^{\mu\nu}b_L F_{\mu\nu} + \text{H.c.}$, the asymmetry is written as

$$\frac{\Gamma(t) - \bar{\Gamma}(t)}{\Gamma(t) + \bar{\Gamma}(t)} = \xi A_t \sin \Delta m_d t \ , \qquad A_t = \frac{2\text{Im}(e^{-i\theta_B}c_7 c_7')}{|c_7|^2 + |c_7'|^2} \ ,$$

where $\Gamma(t)$ ($\bar{\Gamma}(t)$) is the decay width of $B^0(t) \to M_s\gamma$ ($\bar{B}^0(t) \to M_s\gamma$) and M_s is some CP eigenstate ($\xi = +1(-1)$ for a CP even (odd) state) such as $K_S\pi^0$ [7]. $\Delta m_d = 2|M_{12}(B_d)|$ and $\theta_B = \arg M_{12}(B_d)$ where $M_{12}(B_d)$ is the B_d–$\bar{B}_d$ mixing amplitude. Because the asymmetry can be only a few percent in the SM, a sizable asymmetry is a clear signal of new physics beyond the SM.

We calculated the following observables in the FCNC and LFV processes: the CP violation parameter in the K^0–$\bar{K}^0$ mixing ε_K, B_d–$\bar{B}_d$ and B_s–$\bar{B}_s$ mass splittings Δm_d and Δm_s, respectively, A_t and the branching ratios $\text{B}(b \to s\gamma)$, $\text{B}(\mu \to e\gamma)$, $\text{B}(\tau \to \mu\gamma)$ and $\text{B}(\tau \to e\gamma)$. We solved renormalization group equations (RGEs) for Yukawa coupling constants and the SUSY breaking parameters numerically keeping all flavor matrices. After demanding the condition of radiative electroweak symmetry breaking, the free parameters in the minimal supergravity model are the universal scalar mass m_0, the universal gaugino mass M_0, the scalar trilinear parameter A_0, the ratio of two vacuum expectation values $\tan \beta$ and the sign of the Higgsino mass parameter μ. In addition we need to specify neutrino parameters. The phenomenological inputs from neutrino oscillation are two mass-squared differences and the Maki-Nakagawa-Sakata (MNS) matrix. In order to relate these

parameters to f_N and M_ν, we work in the basis for N_i, L_i and E_i where $\tilde{f}_L^{ij} = \hat{f}_L^{ij}$ and $f_N^{ij} = (\hat{f}_N V_L)^{ij}$ ($\hat{f}_L$ and $\hat{f}_N$ are diagonal matrix) at the matching scale M_R. In this basis $\kappa_\nu = V_L^{\mathbf{T}} \hat{f}_N M_\nu^{-1} \hat{f}_N V_L = V_{\mathrm{MNS}}^{0*} \hat{\kappa}_\nu V_{\mathrm{MNS}}^{0\dagger}$ where V_{MNS}^0 is the MNS matrix at M_R and $\hat{\kappa}_\nu$ is a diagonal matrix. Note that although $V_L = V_{\mathrm{MNS}}^{0\dagger}$ when M_ν is diagonal in this basis, two are independent in a general case. Once we specify three neutrino masses, V_{MNS}, V_L and $\hat{f}_N$ we can determine the M_ν matrix. Then using the GUT relation for Yukawa coupling constants, we can calculate all squark and slepton mass matrices through RGEs. Note that V_L essentially determines the flavor mixing in the $\tilde{d}_R$ and $\tilde{l}_L$ sectors in this basis.

As typical examples of the neutrino parameters, we consider the following parameter sets corresponding to (i) the Mikheyev-Smirnov-Wolfenstein (MSW) small mixing angle and (ii) the MSW large mixing angle solutions for the solar neutrino problem [2].

(i) small mixing:

$$m_\nu = \left(2.236 \times 10^{-3},\ 3.16 \times 10^{-3},\ 5.92 \times 10^{-2}\right)\ \mathrm{eV}\ ,$$

$$V_{\mathrm{MNS}} = \begin{pmatrix} 0.999 & 0.0371 & 0 \\ -0.0262 & 0.707 & 0.707 \\ 0.0262 & -0.707 & 0.707 \end{pmatrix}\ ,$$

(ii) large mixing:

$$m_\nu = \left(4.0 \times 10^{-3},\ 5.831 \times 10^{-3},\ 5.945 \times 10^{-2}\right)\ \mathrm{eV}\ ,$$

$$V_{\mathrm{MNS}} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix}\ .$$

In each example we also take M_ν to be proportional to a unit matrix with a diagonal element of $M_R = 4 \times 10^{14}$ GeV so that $V_L = V_{\mathrm{MNS}}^{0\dagger}$ and $\hat{f}_N^{ii} = \sqrt{M_R \hat{\kappa}_\nu^{ii}}$. We fix $m_t^{\mathrm{pole}} = 175$ GeV, $m_b^{\mathrm{pole}} = 4.8$ GeV and the CKM parameters as $V_{cb} = 0.04$, $|V_{ub}/V_{cb}| = 0.08$ and take several values of the phase parameter in the CKM matrix δ_{13} [8]. We take $\tan\beta = 5$ and vary other SUSY parameters m_0, M_0, A_0 and the sign of μ. Various phenomenological

constraints from SUSY particles search are included (for detail see [9]). We also impose $2 \times 10^{-4} < \mathrm{B}(b \to s\,\gamma) < 4.5 \times 10^{-4}$ [11] in the following analysis.

Let us first discuss $\mathrm{B}(\mu \to e\,\gamma)$ and ε_K which turn out to be strong constraints on the parameter space in this model. Fig. 1 shows the correlation between $\mathrm{B}(\mu \to e\,\gamma)$ and $\mathrm{B}(\tau \to \mu\,\gamma)$ for the neutrino parameter set (i) and (ii) for $\delta_{13} = 60°$. We can see that the $\mathrm{B}(\mu \to e\,\gamma)$ becomes a very strong constraint for the case (ii), which is a reflection of the large 1–2 mixing in the V_{MNS} matrix. By requiring $\mathrm{B}(\mu \to e\,\gamma) < 1.2 \times 10^{-11}$ [12], $\mathrm{B}(\tau \to \mu\,\gamma)$ becomes less than 10^{-8} for the case (ii) whereas it can be close to the present experimental bound $(1.1 \times 10^{-6}$ [13]) for the case (i). We also calculated $\mathrm{B}(\tau \to e\,\gamma)$ which turns out to be smaller than 3×10^{-12} in both cases. The constraint from ε_K depends on the parameter δ_{13}. After imposing the $\mathrm{B}(\mu \to e\,\gamma)$ constraint, ε_K can be enhanced by 50% for the case (i) and by a factor of 2 for the case (ii). This means that compared to favorable values in the SM $(50° < \delta_{13} < 90°)$, a smaller value of δ_{13} is allowed due to the extra contributions.

The upper part of Fig. 2 shows a correlation between $\Delta m_s / \Delta m_d$ and $\mathrm{B}(\tau \to \mu\,\gamma)$ for case (i) and $\delta_{13} = 60°$. Here we imposed the constraints from $\mathrm{B}(\mu \to e\,\gamma)$ and ε_K. We also imposed the constraint from Δm_d itself though the deviation of this quantity from the SM value is within 5%. For the theoretical uncertainties we allow $\pm 15\%$ difference for ε_K and $\pm 40\%$ for Δm_d. For $\Delta m_s / \Delta m_d$ we fix the hadronic parameters as $f_{B_s}/f_{B_d} = 1.17$ and $B_{B_s}/B_{B_d} = 1$. We can see that $\Delta m_s / \Delta m_d$ can be enhanced up to 30% compared to the SM prediction. This feature is quite different from the minimal supergravity model without the GUT and right-handed neutrino interactions [9] where $\Delta m_s / \Delta m_d$ is almost the same as the SM value. A_t for the same parameter set is shown as a function of $\mathrm{B}(\tau \to \mu\,\gamma)$ in the lower part of Fig. 2. We can see that $|A_t|$ can be close to 30% when $\mathrm{B}(\tau \to \mu\,\gamma)$ is larger than 10^{-8}. The large asymmetry arises because the renormalization effect due to f_N induces sizable contribution to c_7' through gluino–$\tilde{d}_R$ loop diagrams. The corresponding figure to Fig. 2 for the case (ii) shows that $\mathrm{B}(\tau \to \mu\,\gamma)$ is cut off below 10^{-8} and the maximal deviation of $\Delta m_s / \Delta m_d$ from SM is within 6% and $|A_t|$ becomes at most 6%.

In Fig. 3 we show $\Delta m_s / \Delta m_d$ for several values of δ_{13} for the case (i) and (ii). In these

figures we impose $B(\mu \to e\,\gamma)$ constraint. Thin vertical lines correspond to the case without the experimental constraints from ε_K, Δm_d and the lower bound for $\Delta m_s/\Delta m_d$ [10], and the thick lines are allowed range with these constraints. The allowed range of the SM is also shown in these figures. Because the new contributions to B_d–$\bar{B}_d$ amplitude is small, the time-dependent CP asymmetry of $B \to J/\psi\,K_S$ in this model is essentially the same as the SM value, therefore we can obtain information on δ_{13} once this asymmetry is measured experimentally. For example the asymmetry of $B \to J/\psi\,K_S$ mode is 0.4 for $\delta_{13} = 25°$ and 0.65 for $\delta_{13} = 75°$. This figure means the possible deviation from the SM may be seen in both cases once the CP asymmetry of $B \to J/\psi\,K_S$ mode and $\Delta m_s/\Delta m_d$ are measured.

In our example we took M_R which corresponds to the upper bound of f_N because a larger M_R would lead to the blow-up of f_N below the Planck scale. If we take a lower value of M_R the flavor changing amplitudes essentially scale as M_R.

Finally we would like to comment on genarizations of our result. Firstly for a large $\tan\beta$ case, the constraint from $B(\mu \to e\,\gamma)$ becomes stronger. As a result we cannot see any deviation from the SM for the large mixing case in the figures corresponding to Fig. 2 and 3 for $\tan\beta = 30$, whereas the result is similar for the small mixing case. Secondly we considered the vacuum oscillation case. We see that the pattern of the deviation from the SM is similar to the small mixing case because the effect of the flavor mixing in 1–2 generations turns out to be suppressed by the degeneracy of the two light neutrino masses. In this letter we took the case where M_ν is proportional to a unit matrix. When we consider a more general case, V_L is not necessarily equal to $V_{\mathrm{MNS}}^{0\dagger}$. In addition, the superpotential (1) does not lead to the realistic fermion mass relation especially for the first and the second generations. In order to solve this problem we may have to introduce more free parameters. Although the precise values of the predictions depend on the detail of the model in question, the possible new physics signals such as $B(\tau \to \mu\,\gamma)$, B_s–$\bar{B}_s$ mixing and A_t may be expected as long as some of the neutrino Yukawa coupling constants are large. Because these signals provide quite different signatures compared to the SM and the minimal supergravity model without GUT and right-handed neutrino interactions, future experiments in B physics and

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LFV can provide us important clues on the interactions at very high energy scale.

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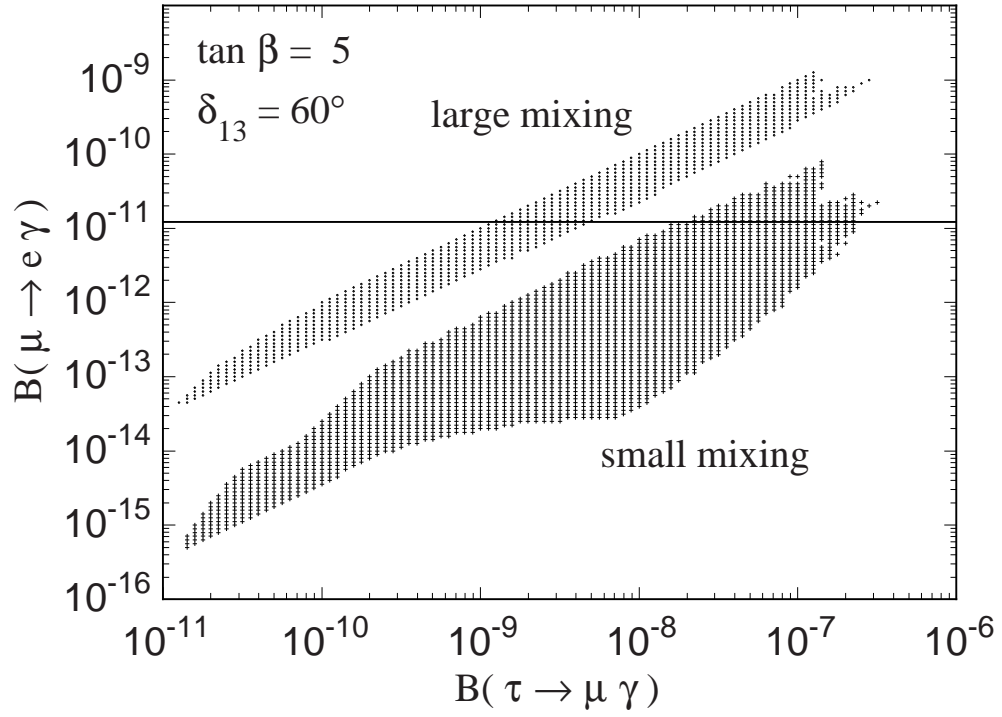FIG. 1.   The correlation between $B(\mu \to e\,\gamma)$ and $B(\tau \to \mu\,\gamma)$ for small and large mixing angle cases. See text for the input parameters.
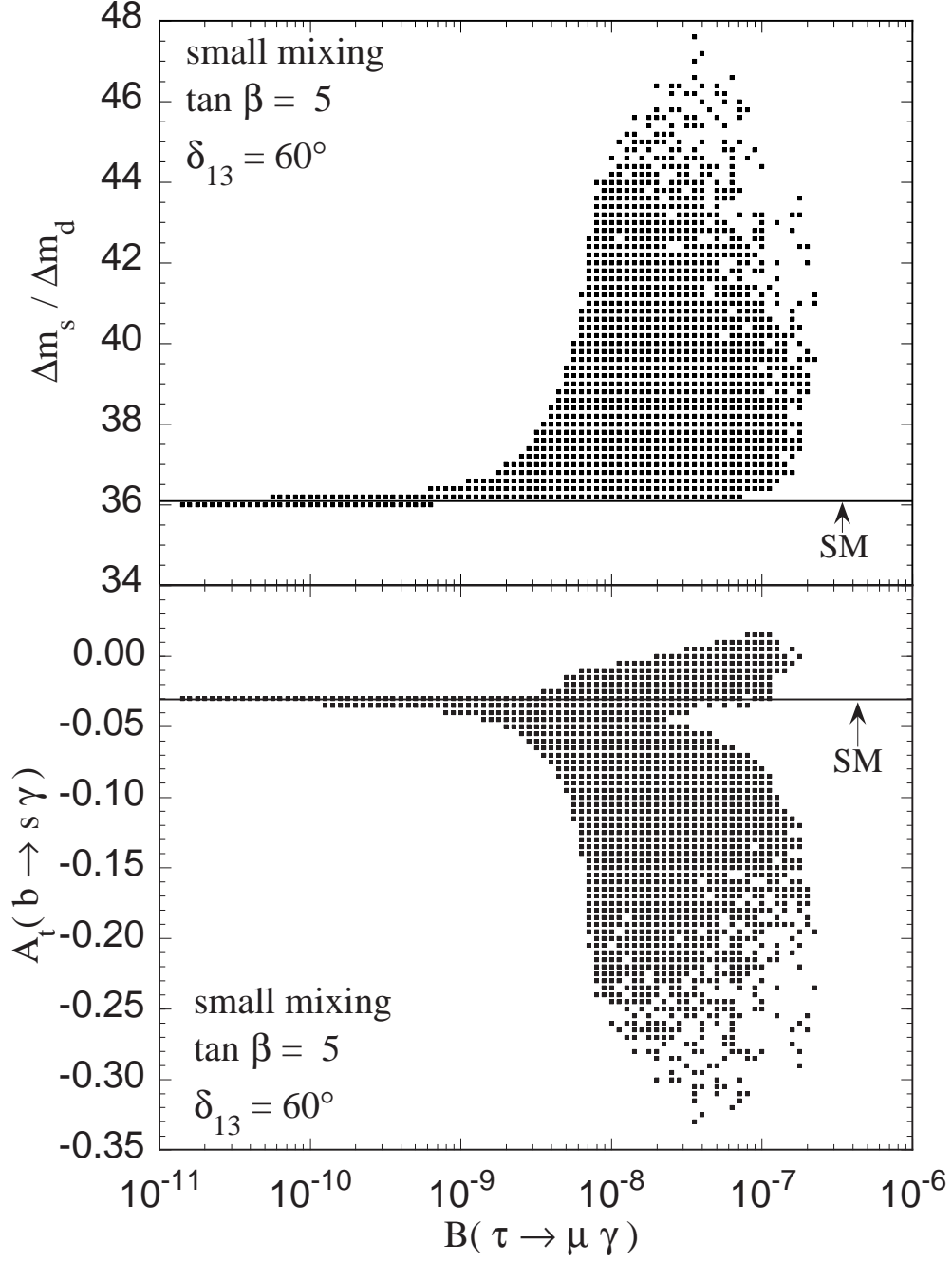
FIG. 2. The ratio of B_s–$\bar{B}_s$ and B_d–$\bar{B}_d$ mass splittings $\Delta m_s/\Delta m_d$ and the magnitude factor A_t of the time-dependent CP asymmetry in the $B \to M_s \gamma$ process as a function of $B(\tau \to \mu \gamma)$ for the small mixing case (i).
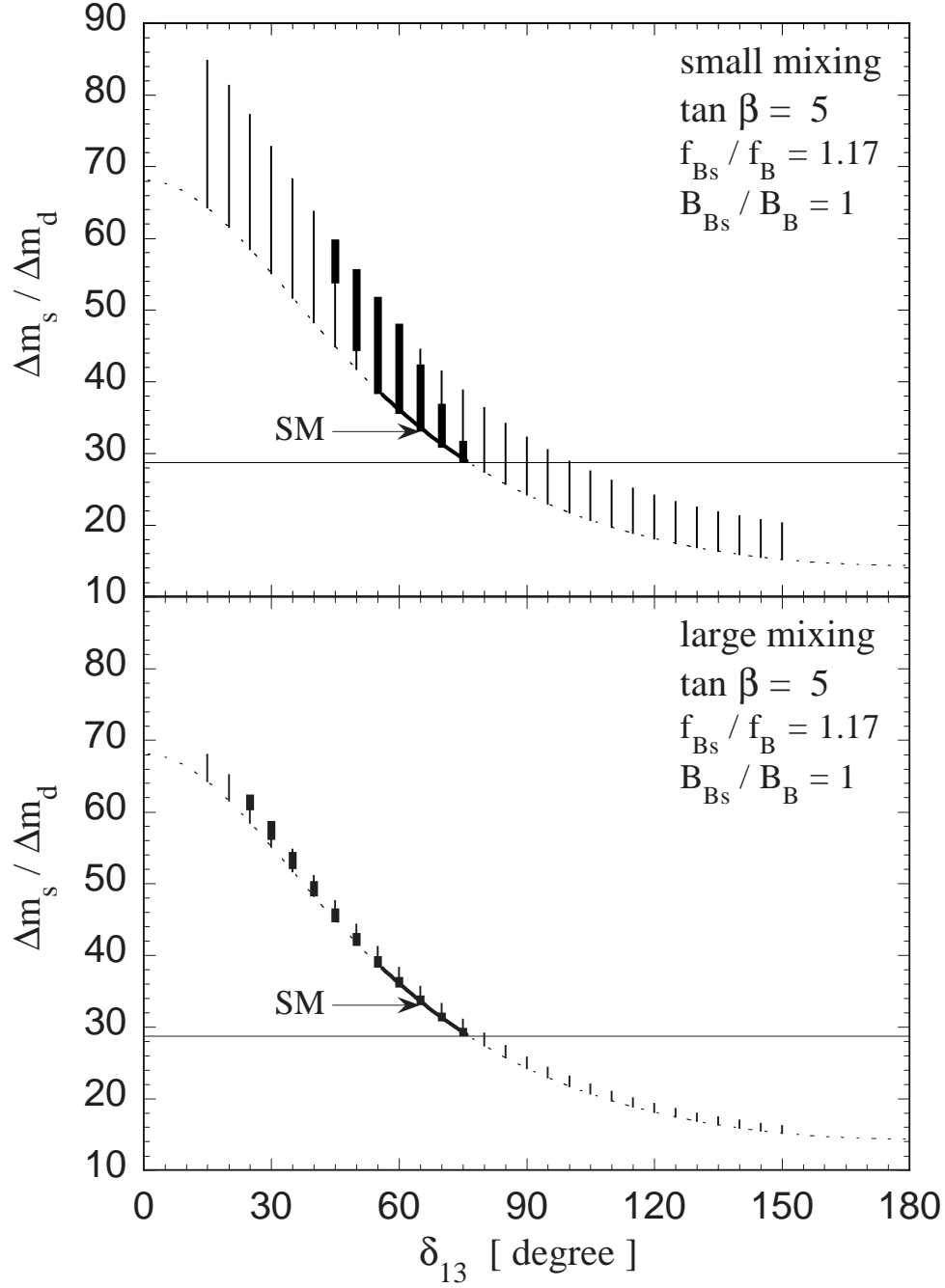
FIG. 3. Possible range of $\Delta m_s/\Delta m_d$ as a function of δ_{13} for small and large mixing cases. Each thick vertical lines shows allowed range with the experimental constraints (see text). Thin vertical lines correspond to the case without the constraints from ε_K, Δm_d and $\Delta m_s/\Delta m_d$. Horizontal line shows the experimental lower bound of $\Delta m_s/\Delta m_d$. The dotted line corresponds to the SM value and the solid section shows the allowed range in the SM.